Chapter 5

Understanding Interest Rates
Present Value

• A dollar paid to you one year from now is less valuable than a dollar paid to you today

• Why?
  – A dollar deposited today can earn interest and become $1 \times (1+i)$ one year from today.
Discounting the Future

Let $i = .10$

In one year \[ $100 \times (1 + 0.10) = $110 \]
In two years \[ $110 \times (1 + 0.10) = $121 \]
or \[ 100 \times (1 + 0.10)^2 \]
In three years \[ $121 \times (1 + 0.10) = $133 \]
or \[ 100 \times (1 + 0.10)^3 \]

In $n$ years
\[ $100 \times (1 + i)^n$ \]
Simple Present Value

\[ PV = \text{today's (present) value} \]
\[ CF = \text{future cash flow (payment)} \]
\[ i = \text{the interest rate} \]

\[ PV = \frac{CF}{(1 + i)^n} \]
Cannot directly compare payments scheduled in different points in the time line

\[
\begin{align*}
\text{Year} & \quad 0 & 1 & 2 & \quad n \\
\text{PV} & \quad 100 & \frac{100}{1+i} & \frac{100}{(1+i)^2} & \frac{100}{(1+i)^n}
\end{align*}
\]
Four Types of Credit Market Instruments

- Simple Loan
- Fixed Payment Loan
- Coupon Bond
- Discount Bond
Yield to Maturity

- The interest rate that equates the present value of cash flow payments received from a debt instrument with its value today
Simple Loan

PV = amount borrowed = $100
CF = cash flow in one year = $110

\[ n = \text{number of years} = 1 \]

\[ \frac{110}{(1 + i)^1} \]

\[ (1 + i) \times 100 = 110 \]

\[ (1 + i) = \frac{110}{100} \]

\[ i = 0.10 = 10\% \]

For simple loans, the simple interest rate equals the yield to maturity.
Fixed Payment Loan

The same cash flow payment every period throughout the life of the loan

LV = loan value

FP = fixed yearly payment

n = number of years until maturity

\[ LV = \frac{FP}{1 + i} + \frac{FP}{(1 + i)^2} + \frac{FP}{(1 + i)^3} + \ldots + \frac{FP}{(1 + i)^n} \]
Coupon Bond

Using the same strategy used for the fixed-payment loan:

\[ P = \text{price of coupon bond} \]

\[ C = \text{yearly coupon payment} \]

\[ F = \text{face value of the bond} \]

\[ n = \text{years to maturity date} \]

\[
P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}
\]
When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate.

The price of a coupon bond and the yield to maturity are negatively related.

The yield to maturity is greater than the coupon rate when the bond price is below its face value.

**Table 1** Yields to Maturity on a 10% Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

<table>
<thead>
<tr>
<th>Price of Bond ($)</th>
<th>Yield to Maturity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,200</td>
<td>7.13</td>
</tr>
<tr>
<td>1,100</td>
<td>8.48</td>
</tr>
<tr>
<td>1,000</td>
<td>10.00</td>
</tr>
<tr>
<td>900</td>
<td>11.75</td>
</tr>
<tr>
<td>800</td>
<td>13.81</td>
</tr>
</tbody>
</table>
Consol or Perpetuity

- A bond with no maturity date that does not repay principal but pays fixed coupon payments forever

\[ P = \frac{C}{i_c} \]

\[ P_c = \text{price of the consol} \]

\[ C = \text{yearly interest payment} \]

\[ i_c = \text{yield to maturity of the consol} \]

Can rewrite above equation as this: \[ i_c = \frac{C}{P_c} \]

For coupon bonds, this equation gives the current yield, an easy to calculate approximation to the yield to maturity
Consol or Perpetuity

What is the yield to maturity on a bond that has a price of $2,000 and pays $100 of interest annually forever?

\[ i_c = \frac{C}{P} \]

\[ P = \$2,000 \]

\[ C = \$100 \]

\[ \Rightarrow i_c = 5\% \]
Discount Bond

For any one year discount bond

\[ i = \frac{F - P}{P} \]

\( F \) = Face value of the discount bond

\( P \) = current price of the discount bond

The yield to maturity equals the increase in price over the year divided by the initial price.

As with a coupon bond, the yield to maturity is negatively related to the current bond price.
Rate of Return

The payments to the owner plus the change in value expressed as a fraction of the purchase price

\[ \text{RET} = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t} \]

RET = return from holding the bond from time \( t \) to time \( t + 1 \)

\( P_t \) = price of bond at time \( t \)

\( P_{t+1} \) = price of the bond at time \( t + 1 \)

\( C \) = coupon payment

\[ \frac{C}{P_t} = \text{current yield} = i_c \]

\[ \frac{P_{t+1} - P_t}{P_t} = \text{rate of capital gain} = g \]
Rate of Return and Interest Rates

- The return equals the yield to maturity only if the holding period equals the time to maturity.

- A rise in interest rates is associated with a fall in bond prices, resulting in a capital loss if time to maturity is longer than the holding period.

- The more distant a bond’s maturity, the greater the size of the percentage price change associated with an interest-rate change.
Rate of Return and Interest Rates (cont’d)

- The more distant a bond’s maturity, the lower the rate of return the occurs as a result of an increase in the interest rate.
- Even if a bond has a substantial initial interest rate, its return can be negative if interest rates rise.
Table 2  One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

<table>
<thead>
<tr>
<th>(1) Years to Maturity When Bond Is Purchased</th>
<th>(2) Initial Current Yield (%)</th>
<th>(3) Initial Price ($)</th>
<th>(4) Price Next Year* ($)</th>
<th>(5) Rate of Capital Gain (%)</th>
<th>(6) Rate of Return (2 + 5) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>10</td>
<td>1,000</td>
<td>503</td>
<td>−49.7</td>
<td>−39.7</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>1,000</td>
<td>516</td>
<td>−48.4</td>
<td>−38.4</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>1,000</td>
<td>597</td>
<td>−40.3</td>
<td>−30.3</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>1,000</td>
<td>741</td>
<td>−25.9</td>
<td>−15.9</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>1,000</td>
<td>917</td>
<td>−8.3</td>
<td>+1.7</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>1,000</td>
<td>1,000</td>
<td>0.0</td>
<td>+10.0</td>
</tr>
</tbody>
</table>

*Calculated with a financial calculator using Equation 3.
Interest-Rate Risk

- Prices and returns for long-term bonds are more volatile than those for shorter-term bonds
- There is no interest-rate risk for any bond whose time to maturity matches the holding period
Real and Nominal Interest Rates

• Nominal interest rate makes no allowance for inflation

• Real interest rate is adjusted for changes in price level so it more accurately reflects the cost of borrowing

• Ex ante real interest rate is adjusted for expected changes in the price level

• Ex post real interest rate is adjusted for actual changes in the price level
Fisher Equation

\[ i = i_r + \pi^e \]

- \( i \) = nominal interest rate
- \( i_r \) = real interest rate
- \( \pi^e \) = expected inflation rate

When the real interest rate is low, there are greater incentives to borrow and fewer incentives to lend. The real interest rate is a better indicator of the incentives to borrow and lend.
**FIGURE 1** Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2008

Question?